

1 Continuous Intro

Note 21

(a) Is

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

a valid density function? Why or why not? Is it a valid CDF? Why or why not?

(b) Calculate the PDF $f_X(x)$, along with $\mathbb{E}[X]$ and $\text{Var}(X)$ if the CDF of X is

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{\ell}, & 0 \leq x \leq \ell, \\ 1, & x \geq \ell \end{cases}$$

(c) Suppose X and Y are independent and have densities

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad f_Y(y) = \begin{cases} 1, & 0 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is their joint distribution? (Hint: for parts (c) and (d), we can use independence in much the same way that we did in discrete probability)

(d) Calculate $\mathbb{E}[XY]$ for the X and Y in part (c).

2 Uniform Distribution

Note 21

You have two fidget spinners, each having a circumference of 10. You mark one point on each spinner as a needle and place each of them at the center of a circle with values in the range $[0, 10)$ marked on the circumference. If you spin both (independently) and let X be the position of the first spinner's mark and Y be the position of the second spinner's mark, what is the probability that $X \geq 5$, given that $Y \geq X$?

3 Darts

Yiming is playing darts. Her aim follows an exponential distribution with parameter 1; that is, the probability density that the dart is x distance from the center is $f_X(x) = \exp(-x)$. The board's radius is 4 units.

- (a) What is the probability the dart will stay within the board?
- (b) Say you know Yiming made it on the board. What is the probability she is within 1 unit from the center?
- (c) If Yiming is within 1 unit from the center, she scores 4 points, if she is within 2 units, she scores 3, etc. In other words, Yiming scores $\lfloor 5 - x \rfloor$, where x is the distance from the center. (This implies that Yiming scores 0 points if she throws it off the board). What is Yiming's expected score after one throw?

4 Darts Again

Note 21

Edward and Khalil are playing darts on a circular dartboard.

Edward's throws are uniformly distributed over the entire dartboard, which has a radius of 10 inches. Khalil has good aim; the distance of his throws from the center of the dartboard follows an exponential distribution with parameter $\frac{1}{2}$.

Say that Edward and Khalil both throw one dart at the dartboard. Let X be the distance of Edward's dart from the center, and Y be the distance of Khalil's dart from the center of the dartboard. What is $\mathbb{P}[X < Y]$, the probability that Edward's throw is closer to the center of the board than Khalil's? Leave your answer in terms of an unevaluated integral.

[Hint: X is not uniform over $[0, 10]$. Solve for the distribution of X by first computing the CDF of X , $\mathbb{P}[X < x]$.]