1 Probabilistic Bounds

Note 17

A random variable X has variance $\text{Var}(X) = 9$ and expectation $\mathbb{E}[X] = 2$. Furthermore, the value of X is never greater than 10. Given this information, provide either a proof or a counterexample for the following statements.

(a) $\mathbb{E}[X^2] = 13$.

(b) $\mathbb{P}[X = 2] > 0$.

(c) $\mathbb{P}[X \geq 2] = \mathbb{P}[X \leq 2]$.

(d) $\mathbb{P}[X \leq 1] \leq 8/9$.

(e) $\mathbb{P}[X \geq 6] \leq 9/16$.

2 Vegas

Note 17

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip their coin. Assume that each person is independently likely to carry a fair or a trick coin.

(a) Let X be the proportion of people whose coin flip results in heads. Find $\mathbb{E}[X]$.

(b) Given the results of your experiment, how should you estimate p? (*Hint:* Construct an unbiased estimator for p using part (a))

(c) How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

3 Working with the Law of Large Numbers

Note 17

(a) A fair coin is tossed multiple times and you win a prize if there are more than 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed multiple times and you win a prize if there are more than 40% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(c) A fair coin is tossed multiple times and you win a prize if there are between 40% and 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(d) A fair coin is tossed multiple times and you win a prize if there are exactly 50% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.