

## 1 Dice Variance

Note 16

(a) Let  $X$  be a random variable representing the outcome of the roll of one fair 6-sided die. What is  $\text{Var}(X)$ ?

(b) Let  $Z$  be a random variable representing the average of  $n$  rolls of a fair 6-sided die. What is  $\text{Var}(Z)$ ?

## 2 Elevator Variance

Note 16

A building has  $n$  upper floors numbered  $1, 2, \dots, n$ , plus a ground floor  $G$ . At the ground floor,  $m$  people get on the elevator together, and each person gets off at one of the  $n$  upper floors uniformly at random and independently of everyone else. What is the *variance* of the number of floors the elevator *does not* stop at?

### 3 Covariance

Note 16

- (a) We have a bag of 5 red and 5 blue balls. We take two balls uniformly at random from the bag without replacement. Let  $X_1$  and  $X_2$  be indicator random variables for the events of the first and second ball being red, respectively. What is  $\text{cov}(X_1, X_2)$ ? Recall that  $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$ .
- (b) Now, we have two bags A and B, with 5 red and 5 blue balls each. Draw a ball uniformly at random from A, record its color, and then place it in B. Then draw a ball uniformly at random from B and record its color. Let  $X_1$  and  $X_2$  be indicator random variables for the events of the first and second draws being red, respectively. What is  $\text{cov}(X_1, X_2)$ ?