1 Head Count

Consider a coin with $\mathbb{P}[\text{Heads}] = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

(a) What is $\mathbb{P}[X = k]$, for some $0 \leq k \leq 20$?

(b) Name the distribution of X and what its parameters are.

(c) What is $\mathbb{P}[X \geq 1]$? Hint: You should be able to do this without a summation.

(d) What is $\mathbb{P}[12 \leq X \leq 14]$?

2 Head Count II

Consider a coin with $\mathbb{P}[\text{Heads}] = 3/4$. Suppose you flip the coin until you see heads for the first time, and define X to be the number of times you flipped the coin.

(a) What is $\mathbb{P}[X = k]$, for some $k \geq 1$?

(b) Name the distribution of X and what its parameters are.

(c) What is $\mathbb{P}[X > k]$, for some $k \geq 0$?

(d) What is $\mathbb{P}[X < k]$, for some $k \geq 1$?

(e) What is $\mathbb{P}[X > k \mid X > m]$, for some $k \geq m \geq 0$? How does this relate to $\mathbb{P}[X > k - m]$?

3 Shuttles and Taxis at Airport

In front of terminal 3 at San Francisco Airport is a pickup area where shuttles and taxis arrive according to a Poisson distribution. The shuttles arrive at a rate $\lambda_1 = 1/20$ (i.e. 1 shuttle per 20 minutes) and the taxis

arrive at a rate $\lambda_2 = 1/10$ (i.e. 1 taxi per 10 minutes) starting at 00:00. The shuttles and the taxis arrive independently.

(a) What is the distribution of the following:

    (i) The number of taxis that arrive between times 00:00 and 00:20?

    (ii) The number of shuttles that arrive between times 00:00 and 00:20?

    (iii) The total number of pickup vehicles that arrive between times 00:00 and 00:20?

(b) What is the probability that exactly 1 shuttle and 3 taxis arrive between times 00:00 and 00:20?

(c) Given that exactly 1 pickup vehicle arrived between times 00:00 and 00:20, what is the conditional probability that this vehicle was a taxi?

(d) Suppose you reach the pickup area at 00:20. You learn that you missed 3 taxis and 1 shuttle in those 20 minutes. What is the probability that you need to wait for more than 10 mins until either a shuttle or a taxi arrives?