

1 Probability Potpourri

Note 13
Note 14

Provide brief justification for each part.

- (a) For two events A and B in any probability space, show that $\mathbb{P}[A \setminus B] \geq \mathbb{P}[A] - \mathbb{P}[B]$.
- (b) Suppose $\mathbb{P}[D | C] = \mathbb{P}[D | \bar{C}]$, where \bar{C} is the complement of C . Prove that D is independent of C .
- (c) If A and B are disjoint, does that imply they're independent?

2 Mario's Coins

Note 14

Mario owns three identical-looking coins. One coin shows heads with probability $1/4$, another shows heads with probability $1/2$, and the last shows heads with probability $3/4$.

(a) Mario randomly picks a coin and flips it. He then picks one of the other two coins and flips it. Let X_1 and X_2 be the events of the 1st and 2nd flips showing heads, respectively. Are X_1 and X_2 independent? Please prove your answer.

(b) Mario randomly picks a single coin and flips it twice. Let Y_1 and Y_2 be the events of the 1st and 2nd flips showing heads, respectively. Are Y_1 and Y_2 independent? Please prove your answer.

(c) Mario arranges his three coins in a row. He flips the coin on the left, which shows heads. He then flips the coin in the middle, which shows heads. Finally, he flips the coin on the right. What is the probability that it also shows heads?

3 Aces

Note 13
Note 14

Consider a standard 52-card deck of cards:

- (a) Find the probability of getting an ace or a red card, when drawing a single card.
- (b) Find the probability of getting an ace or a spade, but not both, when drawing a single card.
- (c) Find the probability of getting the ace of diamonds when drawing a 5 card hand.
- (d) Find the probability of getting exactly 2 aces when drawing a 5 card hand.
- (e) Find the probability of getting at least 1 ace when drawing a 5 card hand.
- (f) Find the probability of getting at least 1 ace or at least 1 heart when drawing a 5 card hand.

4 Balls and Bins

Note 14

Suppose you throw n balls into n labeled bins one at a time.

(a) What is the probability that the first bin is empty?

(b) What is the probability that the first k bins are empty?

(c) Let A be the event that at least k bins are empty. Let m be the number of subsets of k bins out of the total n bins. If we assume A_i is the event that the i th set of k bins is empty. Then we can write A as the union of A_i 's:

$$A = \bigcup_{i=1}^m A_i.$$

Compute m , and use the union bound to give an upper bound on the probability $\mathbb{P}[A]$.

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that “the first bin is empty” and “the first two bins are empty” independent?

(f) Are the events that “the first bin is empty” and “the second bin is empty” independent?