

1 Polynomial Practice

Note 8

(a) If f and g are non-zero real polynomials, how many roots do the following polynomials have at least? How many can they have at most? (Your answer may depend on the degrees of f and g .)

(i) $f + g$

(ii) $f \cdot g$

(iii) f/g , assuming that f/g is a polynomial

(b) Now let f and g be polynomials over $\text{GF}(p)$.

(i) We say a polynomial $f = 0$ if $\forall x, f(x) = 0$. Show that if $f \cdot g = 0$, it is not always true that either $f = 0$ or $g = 0$.

(ii) How many f of degree *exactly* $d < p$ are there such that $f(0) = a$ for some fixed $a \in \{0, 1, \dots, p-1\}$?

(c) Find a polynomial f over $\text{GF}(5)$ that satisfies $f(0) = 1, f(2) = 2, f(4) = 0$. How many such polynomials of degree at most 4 are there?

