

## 1 Propositional Practice

Note 1

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

- (a) There is a real number which is not rational.
- (b) All integers are natural numbers or are negative, but not both.
- (c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.
- (d)  $(\forall x \in \mathbb{Z}) (x \in \mathbb{Q})$
- (e)  $(\forall x \in \mathbb{Z}) (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$
- (f)  $(\forall x \in \mathbb{N}) ((x > 7) \implies ((\exists a, b \in \mathbb{N}) (a + b = x)))$

## 2 Truth Tables

Note 1

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

- (a)  $P \wedge (Q \vee P) \equiv P \wedge Q$
- (b)  $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$
- (c)  $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

### 3 Logical Equivalence?

Note 1

Decide whether each of the following logical equivalences is correct and justify your answer.

(a)  $\forall x (P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$

(b)  $\forall x (P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall x Q(x)$

(c)  $\exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

(d)  $\exists x (P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$